Weak Gravity Conjecture and Holographic/Agegraphic Dark Energy

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Abstract

A criterion that should be satisfied in the inflation and quintessence models has been motivated from the weak gravity conjecture (WGC) by Huang . However, it is found that the criterion is inconsistent with the Holographic dark energy (HDE) and new agegraphic dark energy (NADE) models. In the note, we firstly show that in the HDE and NADE models the criterion should be be replaced respectively by two new criterions. Secondly, we apply the new criterions indicated by WGC to survey the two models. We find that the contradiction between WGC and the NADE model is removed when the new criterion is used. In the HDE model, we find the effects of the spatial curvature and the interaction should be considered in order to match the new criterion.

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1 Introduction

A full quantum theory of gravity has not been known, but it is generally believed that the theory of quantum gravity can be formulated in the context of string theory. Recent progress [1] suggests that there exist a vast number of semi-classical consistent vacua in string theory, named *Landscape* [2]. However, not all semi-classical consistent vacua are actually consistent on the quantum level, and those actually inconsistent vacua are called *Swampland* [3]. Self-consistent

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landscape is surrounded by the swampland. Then, how to select the consistent landscape from the numerous vacua becomes an urgent problem.

Recently, The weak gravity conjecture (WGC) is suggested to be a new criterion to distinguish the landscape from the swampland [4, 5]. The conjecture can be most simply stated as gravity is the weakest force. For a four-dimensional U(1) gauge theory, WGC implies that there is an intrinsic UV cutoff [4]

$$\Lambda \leq gM_p,$$

where g is the gauge coupling constant and M_p is the Planck scale. In [6], the conjecture is applied to the scalar field theory and it is argued that WGC also indicates an intrinsic UV cutoff for the scalar field theories with gravity, e.g.

$$\Lambda \leq \lambda^{1/2} M_p$$

for $\lambda \phi^4$ theory. In the slow-rolling inflation model with the potential $V(\phi) \sim \lambda \phi^4$, it is natural to take the Hubble parameter H as the IR cutoff for the field theory. Then the requirement that the IR cutoff should be lower than the UV cutoff indicates [6]

$$\frac{\lambda^{1/2} \phi^2}{M_p} \sim H \leq \Lambda \leq \lambda^{1/2} M_p, \quad \texttt{or,} \quad \phi \leq M_p \tag{1}$$

This leads Huang in [7] to conjecture that the variation of the inflaton during the period of inflation should be less than M_p,

$$|\Delta \phi| \leq M_p. \tag{2}$$

And it is found that this can make stringent constraints on the spectral index of the inflation model [7].

Furthermore, by arguing that the variation of the canonical quintessence field minimally coupled to gravity should also be less than the Planck scale, the author in [8] used the criterion (2) to explore the quintessence model of dark energy, and found that the theoretic constraints are more stringent than present experiments in some cases [8].

Recently, the criterion (2) has been used to explore the different models of dark energy [9, 10, 11, 12, 13, 14]. By assuming that the holographic dark energy (HDE) scenario [15] is the underlying theory of dark energy and can be described by the low-energy scalar field [16], the criterion is used in [10, 11, 12] to survey the holographic dark energy model. And similarly, the criterion (2) is also used in [13] to survey the new agegraphic dark energy (NADE) [17]. However, it is found in [11] that the criterion (2) cannot be satisfied in the non-interacting HDE mode without spatial curvature. For the interacting HDE model with spatial curvature, it is shown in [12] that it is only on the edge of the parameter space that is possible for WGC to be satisfied. For the NADE model, it is found in [13] that the criterion (2) cannot be satisfied, either.

The tension between WGC and the two important dark energy models makes us nervous. Motivated by this, in the note, we will try to alleviate the tension by suggesting two new criterions indicated by WGC to replace Eq.(2) in the HDE and NADE model respectively.

The note is organized as follows. In the next section, we will first give our motivation, and then suggest two new criterions to replace Eq.(2) in the HDE and NADE models respectively. In Sec.3, we will use the new criterion to survey the ANDE model. In Sec.4, we will use the new criterion to survey the HDE model. In Sec.5, Conclusion and Discussion will be given.

2 New Criterions of WGC in HDE and NADE Models

In the last section, we have recalled how the criterion (2) is motivated in [6, 7]. A key assumption in the motivation is that the IR cutoff scale is taken to be the Hubble parameter, H. In the inflation models and the quintessence models, the assumption is reasonable. However, for the HDE and NADE models, the assumption contradicts the basic motivation of the two models. This can be understood easily. We know that the HDE model arises from the holographic principle [18] which imposes a relation between the UV cutoff and IR cutoff on the effective quantum field theory. Then, motivated by the principle, the energy density of HDE, ρ_d, is taken to be

$$\rho_d = \frac{3c^2 M_p^2}{L^2}, \tag{3}$$

where $M_p = (8\pi G)^{-1/2}$, c is a constant, and L is the IR cutoff length scale which is proposed to be the future event horizon R_h [15]

$$R_h = a(t) \int_t^{+\infty} \frac{dt'}{a(t')}. \tag{4}$$

Then, clearly, in the HDE model, the IR cutoff is not the Hubble scale, but the future event horizon. Actually, it has been pointed out in [19] that the model with $L = H^{-1}$ does not work.

The NADE model can be taken as a different version of the HDE model by choosing L to be the conformal age of the universe. The energy density of NADE, ρ_q, is proposed to be [17]

$$\rho_q = \frac{3n^2 M_p^2}{L^2}, \tag{5}$$

where n is a constant, and the IR cutoff length scale L is taken to be the conformal age of the universe η [17]

$$\eta = \int_0^t \frac{dt'}{a(t')}. \tag{6}$$

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Then in the NADE model, the IR cutoff is also not the Hubble scale, but the conformal age.

Since the Hubble scale can not be taken to be the IR cutoff scale in the HDE and NADE models, we conclude that the criterion (2) cannot be applied to constrain the two models. Then, in order to explore the constraints imposed by WGC on the HDE and NADE models, we need to find a new criterion. To do so, we still follow the analysis of Huang [6, 7]. Considering a slow-rolling quintessence field with the the potential $V(\phi) \sim \lambda\phi^4$, we get an intrinsic UV cutoff indicated by WGC [6]

$$\Lambda \leq \lambda^{-1/2} M_p. \tag{7}$$

Then, by assuming that HDE can be described effectively by the quintessence field, we have

$$\rho_d \simeq V(\phi) \sim \lambda\phi^4 \Rightarrow L^{-2} \sim \frac{\lambda\phi^4}{c^2 M_p^2}. \tag{8}$$

Here the IR cutoff length scale for the scalar field theory should be the future event horizon L as given in Eq.(4). Then the requirement that the IR cutoff should be lower than the UV cutoff tells us that

$$L^{-1} \leq \Lambda \leq \lambda^{-1/2} M_p. \tag{9}$$

Together with Eq.(8), we get

$$|\phi| \leq c M_p. \tag{10}$$

Then from the result, we may naturally conjecture a new criterion to replace Eq.(2) in the HDE model that the variation of the holographic quintessence field should be less than $c M_p$,

$$|\Delta\phi| \leq c M_p. \tag{11}$$

Since $c \geq 1$ in the holographic quintessence model, the new criterion (11) is easier to be satisfied than Eq.(2).

Similarly, for the NADE model, a new criterion can also be motivated that the variation of the agegraphic quintessence field should be less than $n M_p$,

$$|\Delta\phi| \leq n M_p. \tag{12}$$

Obviously, the new criterion (12) is much looser than Eq.(2), since the analysis of the observational data tells us that $n > 2.6$ [20, 21]. Below we will use the criterions (11) and (12) to survey the HDE and NADE models respectively.

3 Agegraphic Quintessence Model and New Criterion of WGC

In the section, we will use the criterion (12) to survey the NADE model. The fractional energy density of NADE is given by

$$\Omega_q = \frac{n^2}{H^2 \eta^2}, \tag{13}$$

where $H = \dot{a}/a$, $a(t)$ is the scale factor in the Friedmann-Robertson-Walker (FRW) metric and a dot denotes the derivative with respect to the cosmic time t. Then using Eqs.(5), (6), (13) and the conservation law of NADE

$$\dot{\rho}_q + 3H(1 + w_q)\rho_q = 0, \tag{14}$$

we obtain the equation of state parameter of NADE [17]

$$w_q = -1 + \frac{2}{3n} \frac{\sqrt{\Omega_q}}{a}. \tag{15}$$

For a flat FRW universe filled by the NADE and the pressureless matter, the Friedmann equation reads

$$H^2 = \frac{1}{3M_p^2}(\rho_q + \rho_m), \tag{16}$$

where ρ_m is the energy density of matter with the conservation law

$$\dot{\rho}_m + 3H\rho_m = 0. \tag{17}$$

Then, from Eq.(13) and using Eqs.(14), (27) and (17), we can get the evolving equation of Ω_q [17]

$$\Omega_q' = -\Omega_q(1 - \Omega_q)\left(\frac{3}{1+z} - \frac{2}{n}\sqrt{\Omega_q}\right), \tag{18}$$

where $\Omega_q' \equiv \frac{d\Omega_q}{dz}$, and $z = \frac{a_0}{a} - 1$ is the cosmological redshift. In the note, the subscript 0 denotes the present value of the corresponding parameter, and we take $a_0 = 1$.

Now considering a single-scalar-field quintessence model with the potential $V(\phi)$, we assume that the field is spatially homogeneous. Then the energy density and pressure of the quintessence scalar field are

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{19}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{20}$$

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Then we can obtain easily

$$\rho_\phi = \frac{\dot{\phi}^2}{1 + w_\phi}, \tag{21}$$

where $w_\phi = \frac{p_\phi}{\rho_\phi}$. Without loss of generality, we may assume $dV/d\phi < 0$ and $\dot{\phi} > 0$. Thus from Eq.(21), we may have

$$\dot{\phi} = \sqrt{(1 + w_\phi)\rho_\phi} \tag{22}$$

Then the criterion (11) tells us

$$c \geq \frac{|\Delta\phi(z)|}{M_p} = \int \frac{\dot{\phi}}{M_p} dt = \int_0^z \sqrt{3[1 + w_\phi(z')]\Omega_\phi(z')} \frac{dz'}{1 + z'}, \tag{23}$$

where $\Omega_\phi = \rho_\phi/(3H^2 M_p^2)$. And the criterion (12) tells us

$$n \geq \frac{|\Delta\phi(z)|}{M_p} = \int \frac{\dot{\phi}}{M_p} dt = \int_0^z \sqrt{3[1 + w_\phi(z')]\Omega_\phi(z')} \frac{dz'}{1 + z'}, \tag{24}$$

By assuming NADE can be describe effectively by the quintessence field, we can have

$$\rho_\phi = \rho_q \Rightarrow \Omega_\phi = \Omega_q, \quad w_\phi = w_q. \tag{25}$$

By substituting the equations into Eq.(24), finally we obtain the constraint imposed by WGC on NADE

$$n \geq \frac{|\Delta\phi(z)|}{M_p} = \int_0^z \sqrt{\frac{2\Omega_q^{3/2}}{n(1 + z)}} dz, \tag{26}$$

where Eq.(15) has been used. Since Ω_q can be obtained by solving Eq.(18) numerically with the initial condition $\Omega_q(z_i) = \frac{n^2}{4(1+z_i)^2}$ at $z_i = 2000$ [17], it is easy for us to check whether NADE is consistent with the condition (26). We display the results in Fig.1. It should be noted that the NADE model has been constrained strictly by using the latest observational data. By analyzing the observational data, it is shown in [20] that $n = 2.807^{+0.087}_{-0.086}$ at the 68.3% confidence level, and $n = 2.807^{+0.176}_{-0.170}$ at the 95.4% confidence level. And it is shown in [21] that $n = 2.886^{+0.084}_{-0.082}$ at 1σ confidence level, and $n = 2.886^{+0.169}_{-0.163}$ at 2σ confidence level. Then from Fig.1, we know that, in the NADE model, the new criterion (12) is consistent with the current observational constraints.

4 Holographic Quintessence Model and New Criterion of WGC

Since the HDE model has been analyzed in detail by using the criterion (2) in [12], we can apply the new criterion (11) to constrain the HDE model just by repeating the analysis in [12] simply. Then in the section, we will only display the results summarily.
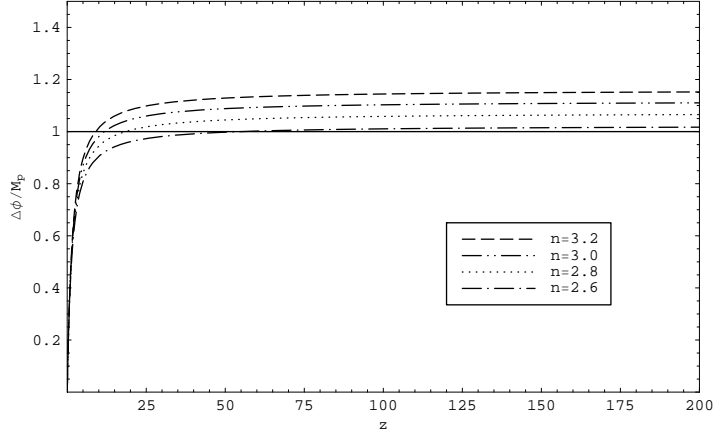
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Fig. 1: $\Delta\phi(z)/M_p$ versus the redshift z in the agegraphic quintessence model with the initial condition $\Omega_q(z_i) = \frac{n^2}{4(1+z_i)^2}$ at $z_i = 2000$. Obviously, $n \geq \Delta\phi(z)/M_p$ holds.

Considering the FRW universe filled by HDE and pressureless matter, the corresponding Friedmann equation reads

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2}(\rho_d + \rho_m), \tag{27}$$

where k is the curvature parameter with $k = -1, 0, 1$ corresponding to a spatially open, flat and closed universe respectively, and ρ_m is the energy density of matter. Usually, by defining

$$\Omega_k = \frac{\rho_k}{\rho_c} = \frac{k}{H^2 a^2}, \quad \Omega_d = \frac{\rho_d}{\rho_c}, \quad \Omega_m = \frac{\rho_m}{\rho_c}, \tag{28}$$

where $\rho_k = k/a^2$ and $\rho_c = 3M_p^2 H^2$, we can rewrite the Friedmann equation as

$$1 + \Omega_k = \Omega_d + \Omega_m. \tag{29}$$

In the universe with the spatial curvature, the future event horizon R_h should be defined as $R_h(t) = a(t)r(t)$ and $r(t)$ satisfying [23]

$$\int_0^{r(t)} \frac{dr}{\sqrt{1 - kr^2}} = \int_t^\infty \frac{dt'}{a(t')}. \tag{30}$$

By defining the effective equation of state parameter as $\dot\rho_d + 3H(1 + w_d^{\text{eff}})\rho_d = 0$, we may get [12]

$$w_d^{\text{eff}} = -\frac{1}{3}\left(1 + 2\sqrt{\frac{\Omega_d}{c^2} - \Omega_k}\right). \tag{31}$$

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The interaction between HDE and matter can be described by the conservation laws

$$\dot{\rho}_m + 3H\rho_m = Q, \tag{32}$$

$$\dot{\rho}_d + 3H(1 + w_d)\rho_d = -Q, \tag{33}$$

where Q denotes the phenomenological interaction term. We, following Ref.[22], consider three types of interaction

$$Q_1 = -3b\rho_d \tag{34}$$

$$Q_2 = -3b(\rho_d + \rho_m) \tag{35}$$

$$Q_3 = -3b\rho_m \tag{36}$$

For convenience, we uniformly express the interaction term as

$$Q_i = -3bH\rho_c\Omega_i, \tag{37}$$

where $\Omega_i = \Omega_d, 1 + \Omega_k$ and Ω_m, for $i = 1, 2$ and 3, respectively.

Then, by using Eqs.(3), (29), (30), (32), (33) and (37), we get the evolving equations of the interacting HDE with spatial curvature [22]

$$\frac{d\widetilde{H}}{dz} = -\frac{\widetilde{H}}{1+z}\Omega_d\left(\frac{3\Omega_d - \frac{\Omega_{k0}(1+z)^2}{\widetilde{H}^2} - 3 - 3b\Omega_i}{2\Omega_D} - 1 + \sqrt{\frac{\Omega_d}{c^2} - \frac{\Omega_{k0}(1+z)^2}{\widetilde{H}^2}}\right), \tag{38}$$

$$\frac{d\Omega_d}{dz} = -\frac{2\Omega_d(1-\Omega_d)}{1+z}\left(\sqrt{\frac{\Omega_d}{c^2} - \frac{\Omega_{k0}(1+z)^2}{\widetilde{H}^2}} - 1 - \frac{3\Omega_d - \frac{\Omega_{k0}(1+z)^2}{\widetilde{H}^2} - 3 - 3b\Omega_i}{2(1-\Omega_d)}\right), \tag{39}$$

where $\widetilde{H} \equiv \frac{H}{H_0}$.

Similarly to the case of NADE, by assuming that HDE can be described effectively by the quintessence field with potential $V(\phi)$, we can have

$$\rho_\phi = \rho_d \Rightarrow \Omega_\phi = \Omega_d, \quad w_\phi = w_d^{\text{eff}}. \tag{40}$$

Then substituting the equations into Eq.(23) and using Eq.(31), we get the new constraint imposed by WGC on HDE as

$$1 \geq \frac{|\Delta\phi(z)|}{cM_p} = \int_0^z \sqrt{2\left(1 - \sqrt{\frac{\Omega_d(z')}{c^2} - \frac{\Omega_{k0}(1+z')^2}{\widetilde{H}^2}}\right)\Omega_d(z')}\frac{dz'}{(1+z')c}. \tag{41}$$

By solving Eqs.(38) and (39) numerically to obtain $\Omega_d(z)$, we can easily check whether the constraint is satisfied in the HDE scenario.
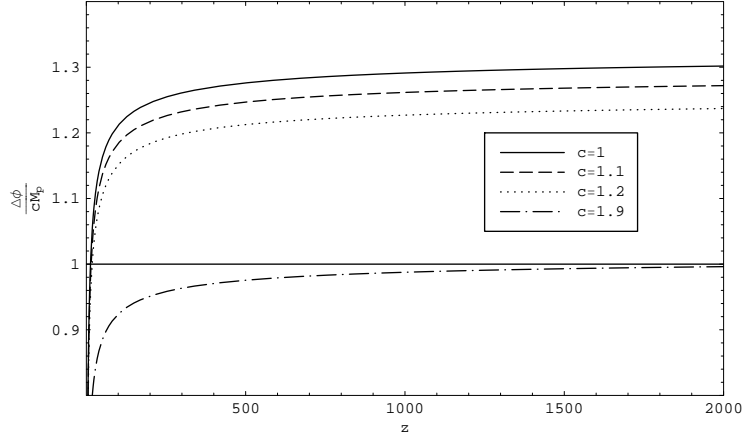
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Fig. 2: $\frac{\Delta\phi(z)}{cM_p}$ versus the redshift z in the non-interacting holographic quintessence model in the flat universe with the fixed initial condition $\Omega_{m0} = 0.34$ for different c.

The result of the HDE model without interaction and spatial curvature is displayed in Fig.2. We find that the new criterion is still inconsistent with the non-interacting HDE model in the flat universe. Naively, in order to match WGC, we should take $c \gtrsim 1.9$ which has been far outside the parameter range obtained in [20]. In Fig.2, we have fixed $\Omega_{m0} = 0.34$, since smaller Ω_{m0} would make it more difficult for Eq.(41) to be satisfied and the value has been bigger than the the maximum value of Ω_{m0} in observational range [20]. Actually, we find that it is only in the HDE model with spatial curvature and interaction $Q = Q_3$ that it is possible for Eq.(41) to be satisfied. Since the conclusion is similar to that in [12], here we only display the result of the HDE model with spatial curvature and interaction $Q = Q_3$ in Fig.3. Roughly, we find that if $b \gtrsim 0.04$ and $\Omega_{k0} \lesssim -0.08$, it is possible to find the combinations of $\Omega_{m0}, \Omega_{k0}, b$ and c which satisfy Eq.(41). The parameter range within which Eq.(11) can be satisfied is larger than that obtained in [12] with Eq.(2).

5 Conclusion and Discussion

Recently, WGC is suggested to be a new criterion to distinguish the landscape from the swampland [4, 5]. From the conjecture, Huang [6, 7] motivates a constraint (2) that should be satisfied in the inflation and quintessence models. However, it is found that Eq.(2) can not be satisfied in the agegraphic quintessence model [13]. And it is very difficult for Eq.(2) to be satisfied in the holographic quintessence model [10, 11, 12]. However, the NADE and HDE models are very successful in explaining the cosmic acceleration and in fitting the observation data. The tension
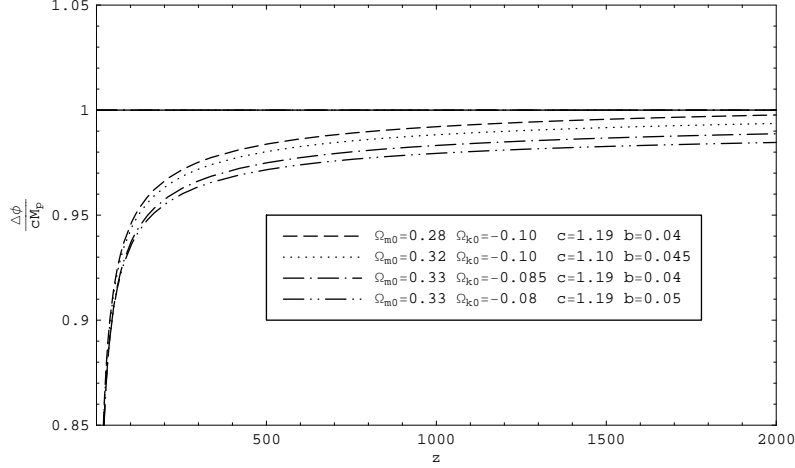
Fig. 3: $\frac{\Delta\phi(z)}{cM_p}$ versus the redshift z in the holographic quintessence model with spatial curvature and interaction term $Q = Q_3$.

between WGC and the two models makes us nervous. However, we find that a key assumption that is used to motivate Eq.(2) contradicts the basic motivation of the HDE and NADE models. Then following the analysis of Huang [6, 7], we find that in the HDE and NADE models, Eq.(2) should be replace by the new criterions (11) and (12) respectively.

Then we apply Eq.(12) to survey the NADE model, and find that the NADE model is consistent with the new criterion (12) within the observational constraint [20, 21]. So the tension between WGC and the NADE model does not exist if the new criterion (12) is used.

However, when applying Eq.(11) to survey the HDE models, we find that WGC cannot be satisfied in the HDE model without interaction and spatial curvature, and it is only in the HDE model with spatial curvature and interaction $Q = Q_3$ that Eq.(11) can be satisfied. The conclusion is similar to what is obtained in [12], but the parameter range within which WGC is satisfied becomes larger. Here we note that the reason why Eq.(11) can not be satisfied in the HDE model with spatial curvature and interaction $Q = Q_1$ or $Q = Q_2$ is that in the two types of models, only the case of $b \leq 0$ is regarded as the relativistic physical situation since positive b will lead ρ_m to be negative in the far future in the two models. At the same time, it can be easily checked that the negative b would make it more difficult for Eq.(41) to be satisfied than the case of $b = 0$. Fortunately, in the model with $Q = Q_3$, there is no such a restriction and we can take $b > 0$. This makes it possible for us to find the combinations of $\Omega_{m0}, \Omega_{k0}, b$ and c satisfying WGC in the model with $Q = Q_3$.

Summarily, in the note we suggest two new criterions to alleviate the tension between WGC and the modes of HDE and NADE. We find that the contradiction between WGC and the NADE

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model is removed when using the new criterion (12). But, in the HDE model, we need involve the effect of the spatial curvature and chose the interaction term as $Q = Q_3$, in order to match the new criterion (11).

Acknowledgments

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